Los Angeles Unified School District
Office of Curriculum Instruction and School Support
2015-2016 Algebra 2 Common Core Curriculum Map

Overview of the Common Core Mathematics Curriculum Map

Introduction to the Document:

Welcome to the Los Angeles Unified School District's Common Core Mathematics Curriculum Map. The mathematics instructional guide for Los Angeles Unified School District is developed as a tool for direction and clarification. It is a living document that is interactive and web-based. There are specific, precise links to provide readily accessible resources needed to appropriately meet the rigors of the common core state standards. The Mathematics Curriculum Map is intended to be a one-stop tool for teachers, administrators, parents, and other school support personnel. It provides information on the Common Core Standards for Mathematics, assessment sample items, and suggested instructional tools organized into units providing one easy-to-read resource.

Components of the Mathematics Curriculum Map:

The Mathematics Curriculum Map is designed around the standards for mathematics k – 12 which are divided into two sets: Practice Standards and Content standards. The Standards for Mathematical Practice are identical for each grade level. They are the expertise and understanding which the mathematics educators will seek to develop in their students. These practices are also the "processes and proficiencies" to be used as instructional "habits of mind" to be developed at all grade levels. It is critical that mathematical literacy is emphasized throughout the instructional process.

The Mathematics Curriculum Map is grouped into four coherent units by grade level. Each unit clarifies the cluster and specific standards students are to master. In addition, the relevant Mathematical Practices and learning progressions are correlated. These sections of the curriculum map define the big idea of the unit. These four units are summarized in the **Unit Organizer** which provides the overview for the year.

Instructional components are specified in:

- **Enduring Understandings** which are the key understandings/big ideas that the students will learn from the unit of study. These are statements that communicate the learning in a way that engages students.
- **Essential Questions** which are based on enduring understandings. They are used to gain student interest in learning and are limited in number. They promote critical or abstract thinking and have the potential of more than one "right" answer. They are connected to targeted standards and are the framework and focus for the unit.
- **Standards**: Targeted (content and skills to be taught and assessed) and supporting (content that is relevant to the unit but may not be assessed; may include connections to other content areas). This includes what students have to know and be able to do (learning targets) in order to meet the standards.

Mathematical literacy is a critical part of the instructional process, which is addressed in:

- **Key Vocabulary** and **Language Goals** which clearly indicate strategies for meeting the needs of EL and SEL students

Planning tools provided are:

- **Instructional Strategies** lead to enduring understandings. They are varied and rigorous instructional strategies to teach content. They are plan experiences that reinforce and enrich the unit while connecting with the standards and assessments. Instructional strategies addresses individual student needs, learner perspectives, integration of technology, learning styles, and multiple intelligences.
- **Resources** and **Performance Tasks** offer concept lessons, tasks, and additional activities for learning.
- **Assessments:** This is also a listing of formative and summative Assessments to guide backwards planning. Student progress in achieving targeted standards/expected learning is evaluated. Entry-level (formative)-based on summative expectations, determine starting points for learning. Benchmark-determine progress of learning, misconceptions, strengths/weaknesses along the learning trajectory.
- **Differentiation** (🕮) falls into three categories:
    - **Universal Design for Learning (UDL) / Universal Access (the approach formerly referred to as Front Loading):** strategies to make the content more accessible to all students, including EL, SEL, Students with Disabilities, and low-achieving students.
    - **Acceleration:** activities to extend the content for all learners, as all learners can have their thinking advanced, and to support the needs of GATE students. These are ideas to deepen the conceptual understanding for advanced learners.
    - **Intervention:** alternative methods of teaching the standards, in which all students can have a second opportunity to connect to the learning, based on their own learning style. They guide teachers to resources appropriate for students needing additional assistance

Using the Mathematics Curriculum Map:

The guide can be thought of as a menu. It cannot be expected that one would do every lesson and activity from the instructional resources provided. To try to teach every lesson or use every activity would be like ordering everything on a menu for a single meal. It is not a logical option. Nor is it possible given the number of instructional days and the quantity of resources. That is why the document is called a "***Mathematics Curriculum Map***" and not a "*Mathematics Pacing Plan*." And, like a menu, teachers select, based on instructional data, which lessons best fit the needs of their students – sometimes students need more time with a concept and at other times, less.

An effective way to use this guide is to review and assess mathematical concepts taught in previous grades to identify potential learning gaps. From there, teachers would map out how much time they feel is needed to teach the concepts within the unit based on the data of their students' needs. For example, some classes may need more time devoted to developing expressions and equations, while another class in the same course may need more focused time on understanding the concept of functions.

The starting point for instructional planning is the standards and how they will be assessed. By first considering how the standards will be assessed, teachers can better select the instructional resources that best build mathematical understanding. There are hundreds of resources available, both publisher- and teacher-created, as well as web-based, that may be used to best teach a concept or skill. Collaborative planning, both within and among courses, is strongly encouraged in order to design effective instructional programs for students.

Learning Progressions:
The Common Core State Standards in mathematics were built on progressions: narrative documents describing the progression of a topic across a number of grade levels, informed both by research on children's cognitive development and by the logical structure of mathematics. The progressions documents can explain why standards are sequenced the way they are, point out cognitive difficulties and pedagogical solutions, and give more detail on particularly knotty areas of the mathematics. This would be useful in teacher preparation and professional development, organizing curriculum, and writing textbooks.

Standards for Mathematical Practice:
The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

The Mathematics Curriculum Map is a living document—it is neither set in stone for all time nor is it perfect. Teachers and other users are encouraged to provide on-going feedback as to its accuracy, usability, and content. Please go to math.lausd.net and share your comments and suggestions. Your participation in making this instructional guide a meaningful and useful tool for all is needed and appreciated.

The grade level Common Core State Standards-aligned Curriculum Maps of the courses in this 2014 edition of the *CCSS Mathematics Curriculum Maps* are the result of the collective expertise of the LAUSD Secondary Mathematics Team.

The District extends its gratitude to the following Algebra 2 curriculum map development team:

Elisa Rose, Andres Flores, Lisa Usher, Reginald Brookens, Jared Dupree, Seng Fong, Kamua Mposi, Geoffrey Buck, Joel Tepper, Ralph Wilkinson, Oksana Pivnenko, Roxanne Klarin, Jack Bloom, Darrell Jones, Rosendo Garcia, Fola Adisa, Hueck Hendrick, Aris Biegler, Norma Alvarez, and Helen Choi. Special thanks to 2014-2015 ESC mathematics team including Amy Aviv, Firoza Kanji, and Stepan Mekhitarian.

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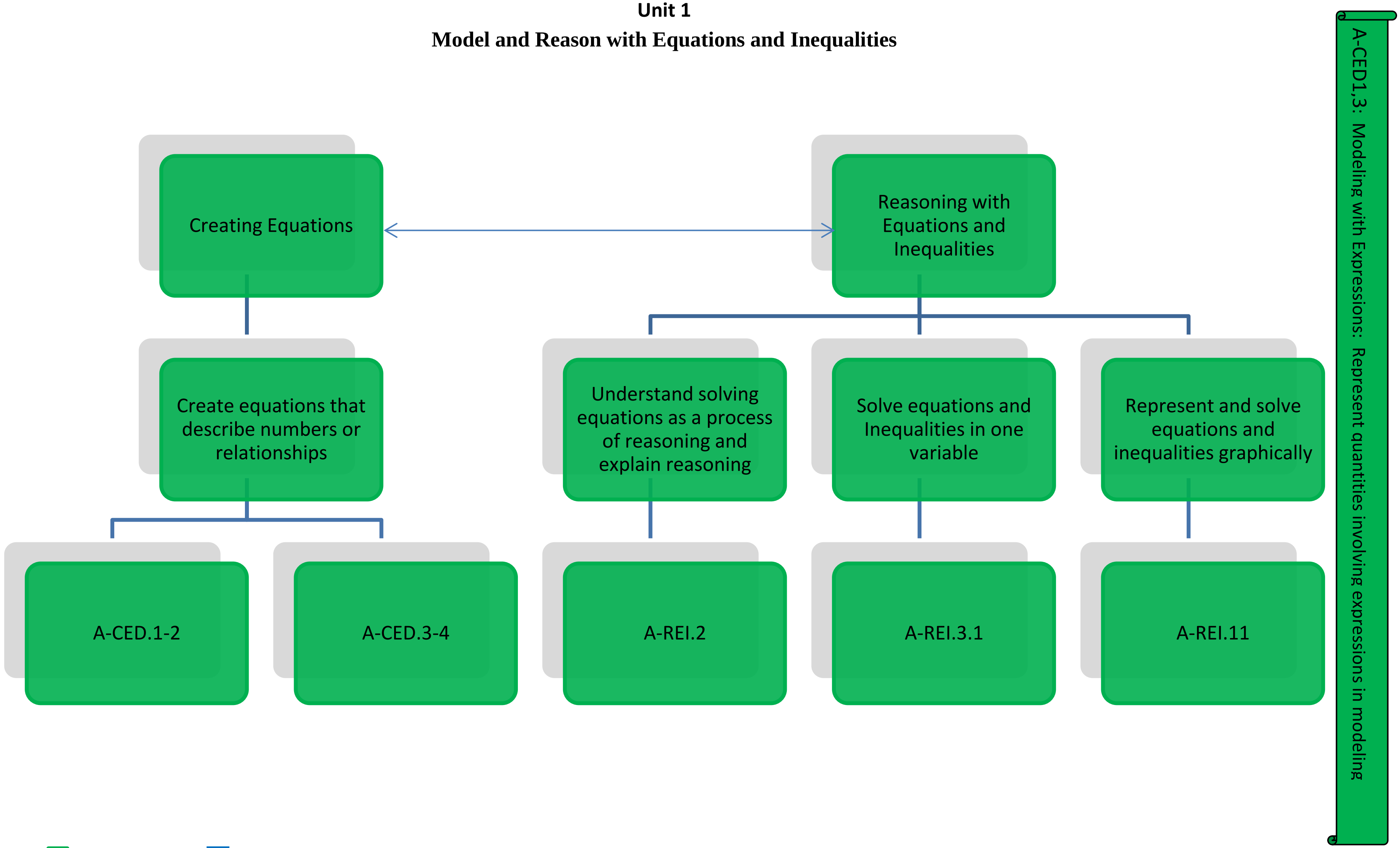
Algebra 2

Unit 1

Model and Reason with Equations and Inequalities

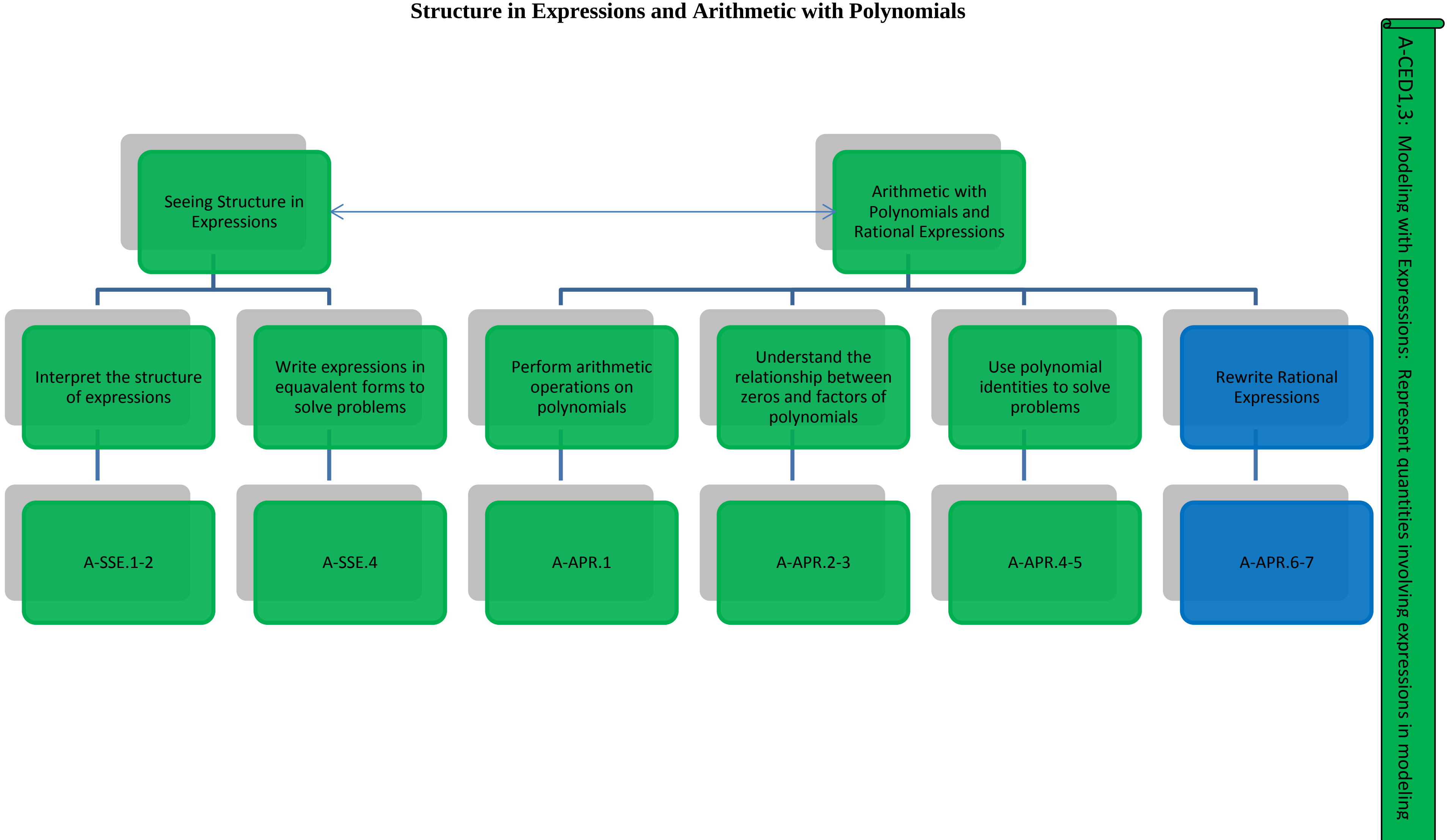


Key: Major Clusters; Supporting Clusters/ Additional Clusters

Algebra 2

Unit 2

Structure in Expressions and Arithmetic with Polynomials

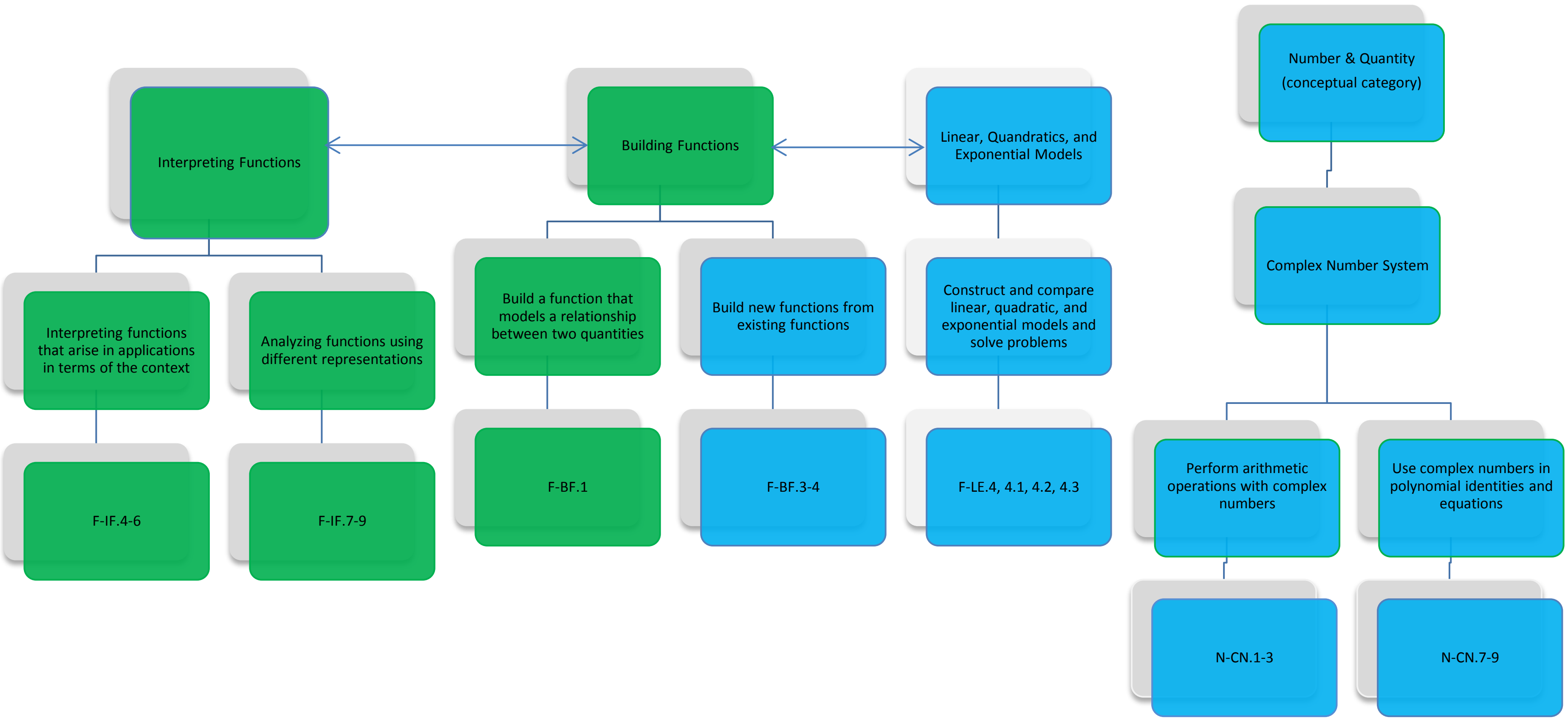


Key: Major Clusters; Supporting Clusters/ Additional Clusters

Algebra 2

Unit 3

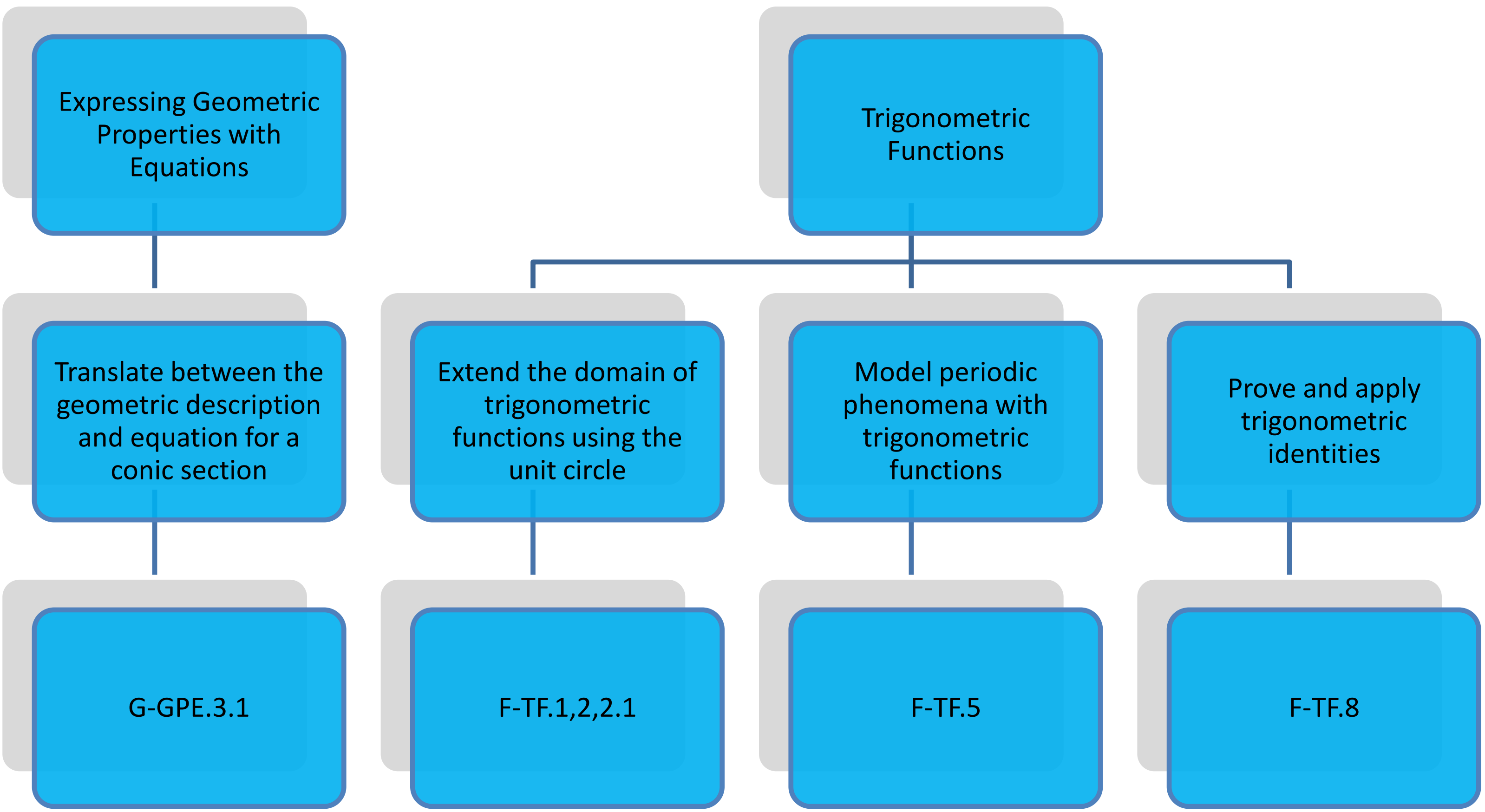
Functions



F-BF.1,3: Moldeling to Build Functions: Build functions that model relationships between quantities

Algebra 2

Unit 4
Geometry and Trigonometry

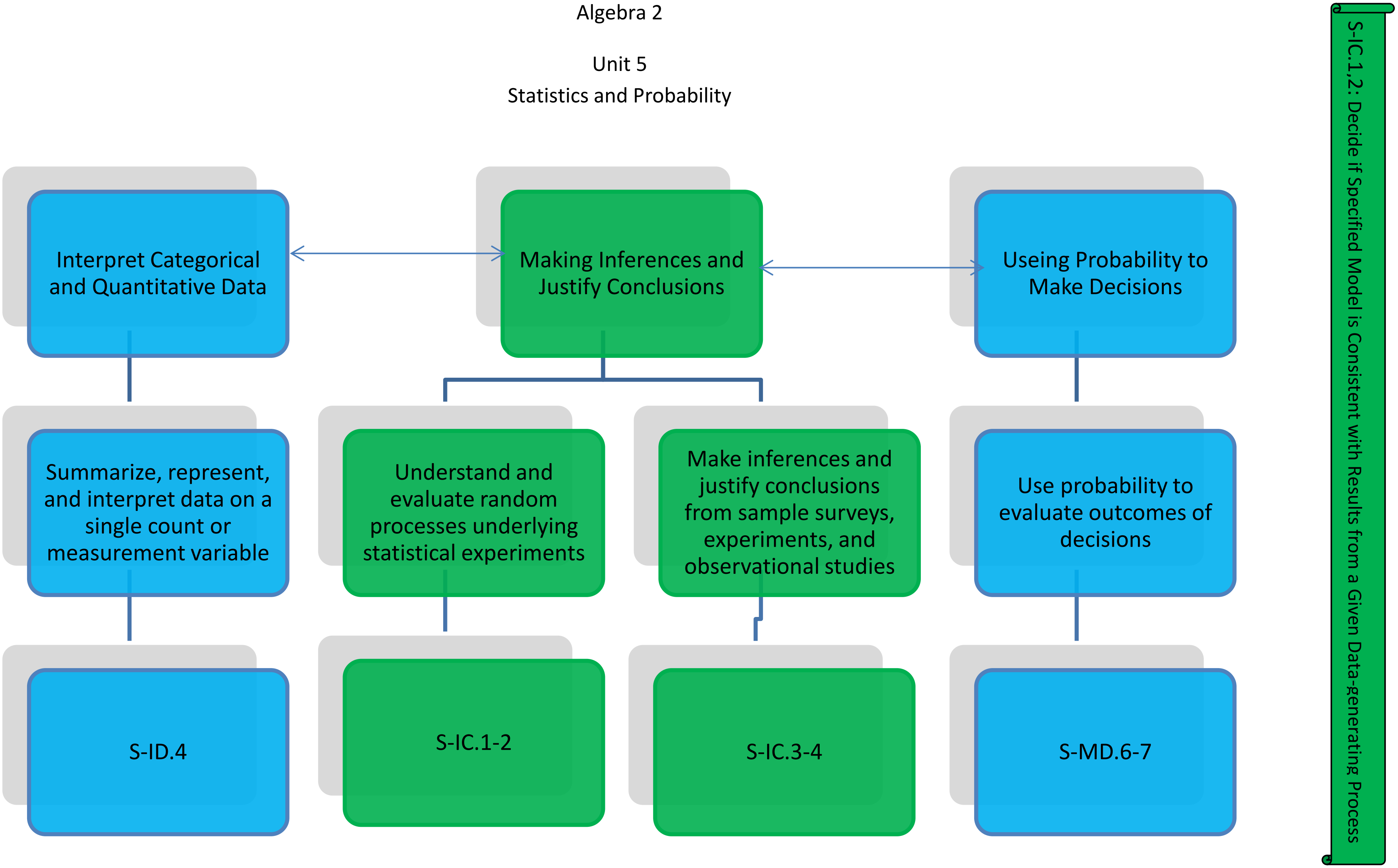


F-TF1,3: Choose Trigonometric functions to Model Periodic Phenomena

Key: Major Clusters; Supporting Clusters/ Additional Clusters

Algebra 2

Unit 5
Statistics and Probability



Key: Major Clusters; Supporting Clusters/ Additional Clusters

Algebra 2 – UNIT 1
Model and Reason with Equations and Expressions

Critical Area: Students use reasoning to analyze equations/ inequalities and develop strategies for solving them. Through reasoning students develop fluency writing, interpreting, analyzing and translating between various forms of linear equations and inequalities. By exploring a question about the world around them (mathematical modeling) and attempting to answer the question students expand the scope of algebraic operations to solve a wide variety of linear and quadratic real world problems. Students explain why the x-coordinates of the points where the graphs $y = f(x)$ and $y = g(x)$ intersects and explore cases involving polynomial, rational, absolute value, exponential, and logarithmic functions.

CLUSTERS	COMMON CORE STATE STANDARDS
(m)Create equations that describe numbers or relationships.	**Algebra – Creating Equations** **A-CED.1**. Create equations and inequalities in one variable **including ones with absolute value** and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. **CA** ★ **A-CED.2**. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★ **A-CED.3**. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. **A-CED.4**. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. ★
(m)Understand solving equations as a process of reasoning and explain the reasoning.	**Algebra – Reasoning with Equations and Inequalities** **A-REI.2.** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
(m)Solve equations and inequalities in one variable.	**A-REI.3.1.** Solve one-variable equations and inequalities involving absolute value, graphing the solutions and interpreting them in context. CA
(m)Represent and solve equations and inequalities graphically.	**A-REI.11.** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★
MATHEMATICAL PRACTICES	
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. **Construct viable arguments and critique**	As you begin the year, it is advised that you start with MP1 and MP 3 and MP4 to set up your expectations of your classroom. This will help you and your students become proficient in the use of these practices. All other practices may be evident based on tasks and classroom activities.

the reasoning of others. **4. Model with mathematics.** 5. Use appropriate tools strategically. 6. Attend to precision. **7. Look for and make use of structure.** 8. Look for and express regularity in repeated reasoning.	

PROGRESSION
Draft High School Progression on Algebra http://opi.mt.gov/PDF/CCSSO/MCCS-MATH/STAGE1/Resources/2012_12-04Draft-High-School-Progression-Algebra.pdf Draft High School Progression on Modeling http://commoncoretools.me/wp-content/uploads/2013/07/ccss_progression_modeling_2013_07_04.pdf

(m) Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.

(S) Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• Different types of relationships between quantities can be modeled with different types of functions. • Graphs are visual representations of solution sets of equations and inequalities. • The arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.	1) What relationships between quantities can be modeled by functions? 2) What does it mean to solve equations graphically? 3) What are the similarities and differences between linear, quadratic, and exponential functions? 4) What do extraneous solutions represent? 5) How does the arithmetic of rational numbers relate to simplifying rational expressions?	• absolute value • constraints • equations • equivalent • exponential • expressions • extraneous • functions • inequalities • Linear • modeling • quadratic • quantities • radical Equations rational equations

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
LAUSD Adopted Textbooks and Programs • Big Ideas Learning - Houghton Mifflin Harcourt, 2015: Big Ideas Algebra 2 • College Preparatory Mathematics, 2013: Core Connections, Algebra 2 • The College Board, 2014:Springboard Algebra 2 **Illustrative Mathematics** • Buying a Car: A-CED.1 http://www.illustrativemathematics.org/illustrations/582 • Basketball: A-CED.1 & A-REI.2 http://www.illustrativemathematics.org/illustrations/702 • How Much Folate: A-CED.2 http://www.illustrativemathematics.org/illustrations/1351 • Dimes and Quarters: A-CED.2 & A-CED.3 http://www.illustrativemathematics.org/illustrations/220 • Growing Coffee: A-CED.3 • http://www.illustrativemathematics.org/illustrations/611 • Bernado and Sylvia Play a Game: A-CED.3 http://www.illustrativemathematics.org/illustrations/1010 • Clea on an Escalator: A-CED.2 http://www.illustrativemathematics.org/illustrations/1003 • Equations and Formulas: A-CED.4 http://www.illustrativemathematics.org/illustrations/393 • Radical Equations: A-REI.2 http://www.illustrativemathematics.org/illustrations/391 • Introduction to Polynomials - College Fund: A-REI.11 http://www.illustrativemathematics.org/illustrations/1551	Most standards in the Creating Equations domain carry a modeling star, denoting their connection with the Modeling category in high school. Therefore mathematical Modeling needs to be at the forefront of conversation with students. For example, equations in high school are also more likely to contain parameters that equations in earlier grades, and so interpreting a solution to an equation might involve more than consideration of a numerical value, but consideration of how the solution behaves as the parameters are varied. Provide examples of real-world problems that can be modeled by writing an equation or inequality. Begin with simple equations and inequalities and build up to more complex equations in two or more variables that may involve quadratic, exponential or rational functions. Give students examples of real-world problems that can be solved by writing an equation, and have students explore the graphs of the equations using technology application to determine which parts of the graph are relevant to the problem context. Provide visual examples of radical and rational equations with technology so that students can see the solution as the intersection of two functions and further understand how extraneous solutions do not fit the model. Have students use technology to graph and explore functions. Discuss the meaning of parameters in the graph including the table, the curves, and the solution to the equation. Have students investigate real-world examples of two-dimensional inequalities. An instructional conversation with all students, in particular English learners will benefit from scaffolds that promote use of academic language. Mathematically Speaking is a scaffold that may be used. http://camsp.net/documents/NCTM-SpeakingArticle.pdf	**Formative Assessment** PARCC – **Graph of Functions** http://www.parcconline.org/sites/parcc/files/HighSchoolAlg2Math3-GraphsofFunctions.pdf **Brett's Race** http://www.parcconline.org/sites/parcc/files/BRHSSampleItem.pdf **LAUSD Assessments** The district will be using the SMARTER Balanced Interim Assessments. Teachers would use the Interim Assessment Blocks (IAB) to monitor the progress of students. Each IAB can be given twice to show growth over time. **State Assessments** California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. There is no assessment for Algebra 1. The 11th grade assessment will include ítems from Algebra 1, Geometry, and Algebra 2 standards. For examples, visit the SMARTER Balance Assessment at: http://www.smarterbalanced.org/ Sample Smarter Balanced Items: http://sampleitems.smarterbalanced.org/itempreview/sbac/index.htm

Illustrative Mathematics • Growth Rate: Given growth charts for the heights of girls and boys, students will use slope to approximate rates of change in the height of boys and girls at different ages. Students will use these approximations to plot graphs of the rate of change of height vs. age for boys and girls. http://illuminations.nctm.org/LessonDetail.aspx?id=L668		

LANGUAGE GOALS for **low achieving, high achieving, students with disabilities and English Language Learners**

Writing:

1) Students will explain and justify the process of solving equations using key vocabulary and multiple representations.
   Example: I solved for the variable by _______. This means that I will multiply 1500 by 3 to get the number of votes Candidate

2) Compare and contrast the differences and similarities between linear, quadratic, and exponential relationships.
   Example:
3) Students will write a summary of the relationship between two variables.
   Example: The rate of change of ____________ and ___________ is _____________.

Listening and Speaking:

1) Students will generate class discussions using specific vocabulary related to solving a wide variety of linear, quadratic, and exponential applications.
2) Students will explain and justify how to solve an equation to a partner as well as restating and summarizing their partner's explanation.
   Example: First I __________ because ____________, second I ___________ because__________,…

Reading:

1) Students will identify the relevant information and details in a passage that help them solve mathematical problems

PERFORMANCE TASKS

Mathematics Assessment Project

- Solving Linear Equations in two Variables: A-CED.2, 3; MP 2,3
  http://map.mathshell.org/materials/lessons.php?taskid=209#task209
- Optimization Problems: Boomerangs: A–CED.2; MP 1,2,3,4
  http://map.mathshell.org/materials/download.php?fileid=1241

Illustrative Mathematics

- Population and Food Supply : A-REI.2, 3, 11 http://www.illustrativemathematics.org/illustrations/645

NCTM Illuminations

- Trout Pond Population: A-CED.2. This investigation illustrates the use of iteration, recursion and algebra to model and analyze a changing fish population. Graphs, equations, tables, and technological tools are used to investigate the effect of varying parameters on the long-term population.

http://illuminations.nctm.org/LessonDetail.aspx?ID=L476

- Exploring Linear Data: A-CED.2. Students model linear data in a variety of settings that range from car repair costs to sports to medicine. Students work to construct scatterplots, interpret data points and trends, and investigate the notion of line of best fit. http://illuminations.nctm.org/LessonDetail.aspx?id=L298

DIFFERENTIATION

UDL/FRONT LOADING	ACCELERATION	INTERVENTION
• Involve students to have a discussion that center around extending their knowledge of creating and analyzing linear equations and inequalities pairs of simultaneous linear equations. Have them use their prior knowledge of graphing linear equations to approaching system of linear and quadratic equations with two variables. • Engage students in an activity that would involve comparing linear equations with quadratics equations, and then quadratics equations and exponential equations. • Have students match linear, quadratic, and exponential functions with their graphs, tables, and equations.	Provide examples of real-world problems that can be modeled by writing linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Have students use technology to graph the functions, make tables of values, or find successive approximations resulting from the function. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Give students examples of real-world problems that can be solved by writing an equation, and have students explore the graphs of the equations using technology application to determine which parts of the graph are relevant to the problem context. Have students write a system of two equations in two variables where one equation is quadratic and the other is linear such that the system has no solution. Explain, using graphs, algebra and/or words, why the system has no solution.	Show students how to create numerical equations and then introduce linear equations in one variables. Students can make comparisons using the numerical and linear equations. For graphing, have students make a T-chart of the equations, graph them and them analyze, find the intersection of the equations, and then explain what that means. Include a case where they would compare simple linear and quadratics equations, e.g. $y=2x$ and $y=x^2$

References:

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8. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx.
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Algebra 2 – UNIT 2
Structure in Expressions and Arithmetic with Polynomials

Critical Area: Students connect the polynomial operations with the background knowledge of the algorithms found in multi-digit integer operations. Students realize that the operations on rational expressions (the arithmetic of rational expressions) are governed by the same rules as the arithmetic of rational numbers. Students analyze the structure in expressions and write them in equivalent forms. By modeling students expand the scope of algebraic operations to solve a wide variety of polynomial equations and real world problems. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The role of factoring, as both an aid to the algebra and to the graphing of polynomials, is explored.

CLUSTERS	COMMON CORE STATE STANDARDS
(m)Interpret the structure of expressions.	**Algebra – Seeing Structure in Expressions** **A-SSE.1.** Interpret expressions that represent a quantity in terms of its context. ★ a. Interpret parts of an expression, such as terms, factors, and coefficients. ★ b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.* ★ **A-SSE.2.** Use the structure of an expression to identify ways to rewrite it.
(m)Write expressions in equivalent forms to solve problems.	**A-SSE.4.** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.* ★
(m)Perform arithmetic operations on polynomials.	**Algebra – Arithmetic with Polynomials and Rational Expressions** **A-APR.1.** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
Understand the relationship between zeros and factors of polynomials.	**A-APR.2.** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$. **A-APR.3.** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Use polynomial identities to solve problems.	**A-APR.4.** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.* **A-APR.5.** Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.**(+)**
Rewrite rational expressions.	**A-APR.6.** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x)$ +

CLUSTERS	COMMON CORE STATE STANDARDS
	$r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. **(+)A-APR.7.** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MATHEMATICAL PRACTICES	
1. Make sense of problems and persevere in solving them. **2. Reason abstractly and quantitatively.** 3. Construct viable arguments and critique the reasoning of others. **4. Model with mathematics.** 5. Use appropriate tools strategically. 6. Attend to precision. **7. Look for and make use of structure.** 8. Look for and express regularity in repeated reasoning.	Emphasize Mathematics Practices 1, 2, 4, and 7 in this unit.

LEARNING PROGRESSION
High School Progression on Algebra http://commoncoretools.me/wp-content/uploads/2013/07/ccss_progression_algebra_2013_07_03.pdf

[1] **Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.**

[2] **Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.**

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• Expressions that represent a quantity in terms of its context can be interpreted and its structure identified and rewritten. • The formula for the sum of a finite geometric series (when the common ratio is not 1) is derived and used to solve problems. • Polynomials form a system analogous to the integers which are closed under the operations of addition, subtraction, and	1) What does the graph of a function represent? 2) How can you represent the zeroes of a function? 3) How can you describe and show the ways you can find the zeroes (roots) of a function? 4) How can the formula for the sum of a finite geometric series be derived and used to solve problems?	• Binomial Theorem • Coefficient • Exponential • Factors • Factorization • Finite • Function • Geometric Series • Infinite • Interpret

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
multiplication and polynomial identities are proven to describe numerical relationships. • Remainder Theorem can be applied for a polynomial p(x). • Zeros of polynomials are identified when suitable factorizations are available and used to construct a rough graph of the function defined by the polynomial. • Binomial Theorem is for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers and known and applied.	5) How can you use the Binomial Theorem to expand powers of expressions?	• Logarithmic • Polynomial • Relation • Remainder Theorem • Terms • Zeros

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
LAUSD Adopted Textbooks and Programs • Big Ideas Learning - Houghton Mifflin Harcourt, 2015: Big Ideas Algebra 2 • College Preparatory Mathematics, 2013: Core Connections, Algebra 2 • The College Board, 2014:Springboard Algebra 2 **Illustrative Mathematics** • Animal Populations: A-SSE.1, 2 http://www.illustrativemathematics.org/illustrations/436 • Sum of Even and Odd: A-SSE.2 http://www.illustrativemathematics.org/illustrations/198 • Seeing Dots: A-SSE.1, 2 http://www.illustrativemathematics.org/illustrations/21 • Zeroes and factorization of a quadratic polynomial I: A-APR.2 http://www.illustrativemathematics.org/illustrations/787 • Zeroes and factorization of a quadratic polynomial II: A-APR.2	• Have students create their own expressions that meet specific criteria (e.g., number of terms factorable, difference of two squares, etc.) and verbalize how they can be written and rewritten in different forms. Additionally, pair/group students to share their expressions and rewrite one another's expressions. • Students may use hands-on or manipulatives, such as algebra tiles, to establish a visual understanding of algebraic expressions and the meaning of terms, factors and coefficients. Technology may be useful to help a student recognize that two different expressions represent the same relationship. • Provide multiple real-world examples of exponential functions. For instance, to illustrate exponential growth, in the equation for the value of an investment over time $A(t) = 15{,}000(1.04)^t$, where the base is 1.04 and is greater than 1; and the $15,000 represents the value of an investment when increasing in value by 4% per year for x years.	**Formative Assessment** Seeing Structure in an Equation http://www.parcconline.org/samples/mathematics/high-school-seeing-structure-equation Seeing Structure in a Quadratic Equation http://www.parcconline.org/samples/mathematics/high-school-seeing-structure-quadratic-equation **LAUSD Assessments** The district will be using the SMARTER Balanced Interim Assessments. Teachers would use the Interim Assessment Blocks (IAB) to monitor the progress of students. Each IAB can be given twice to show growth over time. **State Assessments** California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. There is no assessment for Algebra 1.

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
http://www.illustrativemathematics.org/illustrations/789 • Zeroes and factorization of a non-polynomial function: A-SSE.2 http://www.illustrativemathematics.org/illustrations/796 • Trina's Triangles:A-SSE.4 http://www.illustrativemathematics.org/illustrations/594 • Egyptian Fraction II: A-SSE.6 http://www.illustrativemathematics.org/illustrations/1346		The 11th grade assessment will include ítems from Algebra 1, Geometry, and Algebra 2 standards. For examples, visit the SMARTER Balance Assessment at: http://www.smarterbalanced.org/ Sample Smarter Balanced Items: http://sampleitems.smarterbalanced.org/itempreview/sbac/index.htm

LANGUAGE GOALS for **low achieving, high achieving, students with disabilities and English Language Learners**

- Students will explain orally and in writing how to interpret parts of an expression, such as terms, factors, and coefficients
  Example: I will interpret $P(1 + r)^n$ as ______ P and a ________ not depending on _____.
- Students will discuss how to derive and solve problems with the formula for the sum of a finite geometric series.
- Students will explain how the zeros of polynomials are identified when suitable factorizations are available and construct a rough graph of the function defined by the polynomial.
- Students will orally and in writing explain how to expand $(x + y)^n$ in powers of x and y for a positive integer n.

PERFORMANCE TASKS

Illustrative Mathematics
- Course of Antibiotics: A-SSE.4 http://www.illustrativemathematics.org/illustrations/805
- Cantor Set: A-SSE.4 http://www.illustrativemathematics.org/illustrations/929
- A Lifetime of Savings: A-SSE.4 http://www.illustrativemathematics.org/illustrations/1283
- Combined Fuel Efficiency: A-SSE.6 http://www.illustrativemathematics.org/illustrations/825

Mathematics Assessment Project
Representing Polynomials: A-APR http://map.mathshell.org/materials/download.php?fileid=1271
Interpreting Algebraic Expressions: A-APR http://map.mathshell.org/materials/download.php?fileid=694

LAUSD Mathematics website – http://math.lausd.net
Parabola Activity

DIFFERENTIATION📖		
UDL/FRONT LOADING	ACCELERATION	INTERVENTION
Have students review how to interpret parts of an expression, such as terms, factors, and coefficients. Design an activity for students to practice graphing both linear and simple quadratics equations such as: $y = x^2$.	Shirley and her colleague are trying to develop a simple algorithm for adding consecutive numbers (1, 2, 3,…, n). Her colleague suggested that they can add the numbers by using the following expression (1+n)*n/2. Show how this is possible. Here the students would recognize that this is Gauss rule of adding numbers. Can this relationship or rule be true for adding consecutive odd numbers, such as: 1, 3, 5, … n-1? Could this work for adding any consecutive even numbers, such as 2, 4, 6,…, n+1? Have students prove the equation of a parabola using the Parabola Paper Folding Activity. Show that the sum of *n* odd natural numbers is n^2.	• Use of real context examples to demonstrate the meaning of quadratics equation, such rocket trajectory, basketball path when thrown to the hoop, etc. • Have students use technology, such as graphing calculator, graphing apps, and other software to graph both a linear function and quadratic function on the same plane. Engage them in a discussion to identify the zeros of polynomials and use the zeros to construct a rough graph of the function and discuss what that means. • Provide a situation that uses realia to further demonstrate the meaning of zeros of polynomial function, such as quadratic.

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1. National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics)*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
2. McCallum, W., Zimba, J., Daro, P. (2011, December 26 Draft). *Progressions for the Common Core State Standards in Mathematics*. Cathy Kessel (Ed.). Retrieved from http://ime.math.arizona.edu/progressions/#committee.
3. Engage NY. (2012). New York Common Core Mathematics Curriculum. Retrieved from http://engageny.org/sites/default/files/resource/attachments/a-story-of-ratios-a-curriculum-overview-for-grades-6-8.pdf.
4. Mathematics Assessment Resource Service, University of Nottingham. (2007 - 2012). Mathematics Assessment Project. Retrieved from http://map.mathshell.org/materials/index.php.
5. Smarter Balanced Assessment Consortium. (2012). Smarter Balanced Assessments. Retrieved from http://www.smarterbalanced.org/.
6. Partnership for Assessment of Readiness for College and Career. (2012). PARCC Assessments. Retrieved from http://www.parcconline.org/parcc-assessment.
7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
8. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx.
9. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions.

Algebra 2 – UNIT 3
FUNCTIONS

Critical Area: Instructional time should focus on relating arithmetic of rational expressions to arithmetic of rational numbers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. Students will expand understandings of functions and graphing to include trigonometric functions. Building on their previous work with functions and on their work with trigonometric ratios and circles in the Geometry course, students now use the coordinate plane to extend trigonometry to model periodic phenomena. Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function.

CLUSTERS	COMMON CORE STATE STANDARDS
(m) Interpreting functions that arise in applications in terms of the context **(m) Analyze Functions Using Different Representations**	**Functions – Interpreting Functions** **F-IF.4**. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★ **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. ★ **F-IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★ **F-IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ★ c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. ★ e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. ★ **F-IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **F-IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
(m) Build a function that models a relationship between two quantities **(s) Build new functions from existing functions**	**Functions – Building Functions** **F-BF.1** Write a function that describes a relationship between two quantities.★ b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*★

	F-BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* **F-BF.4** Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x + 1)/(x - 1)$ *for* $x \neq 1$
(s) Construct and compare linear, quadratic, and exponential models and solve problems	**Functions – Linear, Quadratic, and Exponential Models** **F-LE.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. ★[Logarithms as solutions for exponentials.] **4.1 Prove simple laws of logarithms. CA ★** **4.2 Use the definition of logarithms to translate between logarithms in any base. CA ★**
(s) Perform arithmetic operations with complex numbers **(s) Use complex numbers in polynomial identities and equations.** [Polynomials with real coefficients.]	**Number and Quantity – Complex Number System** **N-CN.1.**Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. **N-CN.2**. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **N-CN.7** Solve quadratic equations with real coefficients that have complex solutions. **N-CN.8** (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$. **N-CN.9** (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
MATHEMATICAL PRACTICES	LEARNING PROGRESSIONS
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. Construct viable arguments and critique the reasoning of others. 4. **Model with mathematics.** 5. **Use appropriate tools strategically.** 6. **Attend to precision.** 7. **Look for and make use of structure.** 8. Look for and express regularity in repeated reasoning.	Emphasize Mathematics Practices 1, 2, 4, 5, and 7 in this unit.

LEARNING PROGRESSIONS
High School Progression on Functions http://commoncoretools.me/wpcontent/uploads/2013/07/ccss_progression_functions_2013_07_02.pdf

(m) Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.

(S) Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.
★**Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.**
(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• Functions need to be understood and interpreted in terms of their context. • A function can be represented in different ways; these different representations help with analysis of the function. • A function can be used to model the relationship between two quantities. • New functions from existing functions can be understood and built. • Comparing linear, quadratic and exponential models to solve problems is understood and constructed. • Real and complex numbers are important in solving and understanding polynomial equations.	1. What is a function and how does it model a relationship between two quantities? 2. How would you write a function that describes a relationship between two quantities? 3. What are the differences and similarities between real and complex solutions of polynomial equations? Explain graphically or algebraically. 4. How do you differentiate between an exponential and a logarithmic function? 5. How and when do we use laws of logarithms?	• absolute value function • complex numbers • complex roots • end behavior • function • Exponential • interpret • inverse function • Laws of Logarithms • logarithmic • periodicity • piecewise function • relation • relative Maximums • relative Minimum • step function • symmetries • transformations • trigonometric

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
LAUSD Adopted Textbooks and Programs • Big Ideas Learning - Houghton Mifflin Harcourt, 2015: Big Ideas Algebra 2 • College Preparatory Mathematics, 2013: Core Connections, Algebra 2 • The College Board, 2014:Springboard Algebra 2 **Illustrative Mathematics** Bacteria Populations: F-LE.4 http://www.illustrativemathematics.org/illustrations/370 Running Time: F-IF.7c	Students can make use of graphing software to investigate the effects of replacing a function (x) by $(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for different types of functions (MP.5). Tables and graphs should be used to support student understanding of F-BF-4a. This standard dovetails with standard F-LE-4 and should be taught in progression with it. Students understand logarithms as functions that *undo* their corresponding exponential functions; opportunities for instruction should emphasize	**Formatiive Assessment** PARCC - http://www.parcconline.org/samples/mathematics/high-school-mathematics

http://www.illustrativemathematics.org/illustrations/1539 Graphs of Power Functions: F-IF.7c http://www.illustrativemathematics.org/illustrations/627 Exponentials and Logarithms I: F-BF.4 http://www.illustrativemathematics.org/illustrations/600 Exponentials and Logarithms II: F-BF.5, F-LE.4 http://www.illustrativemathematics.org/illustrations/615 Complex number patterns: N-CN.1 http://www.illustrativemathematics.org/illustrations/722 Powers of a complex number: N-CN.2 http://www.illustrativemathematics.org/illustrations/1689 Completing the square: N-CN.7; A-REI.4 http://www.illustrativemathematics.org/illustrations/1690 **Inside Mathematics** Measuring Mammals- F-BF.4 http://www.insidemathematics.org/problems-of-the-month/pom-measuringmammals.pdf	this relationship.	**LAUSD Assessments** The district will be using the SMARTER Balanced Interim Assessments. Teachers would use the Interim Assessment Blocks (IAB) to monitor the progress of students. Each IAB can be given twice to show growth over time. **State Assessments** California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. There is no assessment for Algebra 1. The 11th grade assessment will include ítems from Algebra 1, Geometry, and Algebra 2 standards. For examples, visit the SMARTER Balance Assessment at: http://www.smarterbalanced.org/ Sample Smarter Balanced Items: http://sampleitems.smarterbalanced.org/itempreview/sbac/index.htm

LANGUAGE GOALS for low achieving, high achieving, students with disabilities and English Language Learners

- Students will describe orally and in writing the process of graphing functions using the terms: intercepts, end behavior, and maximum/minimum.
- Students will describe orally and in writing transformations in terms of the parent function.
- Students will compare and contrast functions and their inverses in oral and written form.
- Students will describe orally and in writing the steps required to express exponential equations in logarithmic form and solve them.
- Student will describe orally and writing how they apply polynomial operations to complex numbers; *for example*, $x^2 + 4 = (x + 2i)(x - 2i)$.

PERFORMANCE TASKS

Mathematics Assessment Project Formative Assessments/Tasks

Patchwork – F-BF.1 http://map.mathshell.org/materials/download.php?fileid=754

Sidewalk Patterns – F-BF.1 http://map.mathshell.org/materials/download.php?fileid=760

Printing Tickets – F-IF.4 http://map.mathshell.org/materials/download.php?fileid=772

Illustrative Mathematics

Identifying graph of functions – F.IF.7c

http://www.illustrativemathematics.org/illustrations/803

Inside Mathematics

Digging Dinosaurs- F-IF.8, F-LE.1 http://www.insidemathematics.org/problems-of-the-month/pom-diggingdinosaurs.pdf

DIFFERENTIATION 🕮		
UDL/FRONT LOADING	ACCELERATION	INTERVENTION
Have students discuss the relationship between a function and definition of its inverse. Engage students in an activity to graph square root, Cube root, and piecewise-defined functions, including step functions and absolute value functions. Have students practice how to graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	Activate a discussion around explicit and implicit function. Students could use a T-chart to describe the differences between explicit and implicit function. Have students show symbolically and graphically how to transform a function to its inverse. Show that when f(x)=2x+3, then x=2f(x)+3. Solving for f(x), the resulting inverse will be f(x)=(x-3)/2 Example: y=2x+3, then x=2y+3 solve for y gives the inverse of (x) as y=(x-3)/2. Graphically you can show the inverse of a function. Make a table of the function f(x) and its inverse. Graph x and f(x) values from the table and then develop another table by switching the numbers.	Student use apps, software, or graphing calculator to practice graphing square root, Cube root, and piecewise-defined functions, including step functions and absolute value functions. They describe the behavior of each of the above functions and write about them.

References:

1. National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics)*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
2. McCallum, W., Zimba, J., Daro, P. (2011, December 26 Draft). *Progressions for the Common Core State Standards in Mathematics*. Cathy Kessel (Ed.). Retrieved from http://ime.math.arizona.edu/progressions/#committee.
3. Engage NY. (2012). New York Common Core Mathematics Curriculum. Retrieved from http://www.engageny.org/sites/default/files/resource/attachments/algebra-ii-m1-module-overview-and-assessments.pdf.
4. Mathematics Assessment Resource Service, University of Nottingham. (2007 - 2012). Mathematics Assessment Project. Retrieved from http://map.mathshell.org/materials/index.php.
5. Smarter Balanced Assessment Consortium. (2012). Smarter Balanced Assessments. Retrieved from http://www.smarterbalanced.org/.
6. Partnership for Assessment of Readiness for College and Career. (2012). PARCC Assessments. Retrieved from http://www.parcconline.org/parcc-assessment.
7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
8. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx.
9. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions.

Algebra 2 – UNIT 4
Geometry and Trigonometry

Critical Area: Students use algebraic manipulation, including completing the square, as a tool for geometric understanding to determine if the equation represents a circle or a parabola. They graph shapes and relate the graphs to the behavior of the functions with the transformation on the variable (e.g. the graph of y=f (x+2). Students expand on their understanding of the trigonometric functions first developed in Geometry to explore the graphs of trigonometric functions with attention to the connection between the unit circle representation of the trigonometric functions and their properties, use trigonometric functions to model periodic phenomena. Students use Pythagorean identity to find the trig function outputs given the angle and understand that interpretation of sine and cosine yield the Pythagorean Identity.

CLUSTERS	COMMON CORE STATE STANDARDS (*) Indicates a modeling standard linking mathematics to everyday life, work, and decision making.
Translate between the geometric description and the equation for a conic section	**Geometry – Expressing Geometry Properties with Equations** **G-GPE.3.1.** Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, use the method for completing the square to put the equation into standard form; identify whether the graph of the equation is a circle, ellipse, parabola, or hyperbola, and graph the equation. [In Algebra II, this standard addresses circles and parabolas only.] CA
Extend the domain of the trigonometric functions using the unit circle **Model periodic phenomena with trigonometric functions** **Prove and apply trigonometric identities**	**Functions – Trigonometric Functions** **F-TF.1.** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **F-TF.2.** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **F-TF.2.1.** Graph all 6 basic trigonometric functions. **F-TF.5.** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★ **F-TF.8.** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant.
MATHEMATICAL PRACTICES	
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. **Construct viable arguments and critique the reasoning of others.**	Emphasize all the mathematical practice standards as you address the standards in this unit. F-TF.5 would provide the opportunity to link mathematics to everyday life, work, and decision making.

4. **Model with mathematics.** 5. **Use appropriate tools strategically.** 6. **Attend to precision.** 7. **Look for and make use of structure.** 8. **Look for and express regularity in repeated reasoning.**	

LEARNING PROGRESSIONS
High School Progression on Functions http://commoncoretools.me/wp-content/uploads/2013/07/ccss_progression_functions_2013_07_02.pdf

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.
(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• A circle is a set of points that can be defined by an equation. This measurement is expressed in radians rather than degrees. • Students extend the domain of trigonometric functions using the unit circle. • Students establish a way to measure angles with respect to arc length. • The trigonometric functions are extended to all real numbers to describe rotations around the unit circle. • Our world is periodic. The amount of sunlight a city receives on a given day, high and low tides are all real life instances where sinusoids explain and model real life phenomena. • Prove the Pythagorean identity $\sin^2(\Theta) + \cos^2(\Theta) = 1$ and use it to find $\sin(\Theta)$, $\cos(\Theta)$, or $\tan(\Theta)$ given $\sin(\Theta)$, $\cos(\Theta)$, or $\tan(\Theta)$ and the quadrant of the angle. • The Theorem of Pythagoras comes up in many places, including trigonometry, through the use of standard graphing form of a circle. It is used to prove many trigonometric identities.	• How do you write the equation of a circle? • What is the angle of rotation, and how is it measured? • How can you explain the unit circle? • Why do we need radian measure? • Why are radians said to be unitless measures of angles? • How can sine, cosine, and tangent functions be defined using the unit circle? • What are periodic functions? • Why is modeling them so important? • Why is the Theorem of Pythagoras so essential?	amplitude angle of rotation completing the square cosecant function cosine function cotangent function coterminal dilation domain initial side intercepted arc midline periodic function phase shift quadrants radian measure range reference angles secant function sine function sinusoid special right triangles standard form of a circle standard position tangent function

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
		terminal side translation unit circle

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
LAUSD Adopted Textbooks and Programs • Big Ideas Learning - Houghton Mifflin Harcourt, 2015: Big Ideas Algebra 2 • College Preparatory Mathematics, 2013: Core Connections, Algebra 2 • The College Board, 2014:Springboard Algebra 2 **California Revised Mathematics Framework:** http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp. **Illustrative Mathematics Resources**: • Explaining the equation for a circle:G-GPE.3 http://www.illustrativemathematics.org/illustrations/1425 • Foxes and Rabbits 3: F-TF.5 http://www.illustrativemathematics.org/illustrations/817 • Trig Functions and the Unit Circle : F-TF.2 https://www.illustrativemathematics.org/illustrations/1820 **NCTM Illuminations** • Graphs from the Unit Circle: F-TF.1, 2 http://illuminations.nctm.org/LessonDetail.aspx?id=L785 **Miscellaneous Sources** Gravel Roads and Sinusoidal Patterns: http://www.nsa.gov/academia/_files/collected_learni	Show students how to find sine, cosine, and tangent values by constructing right triangles on a Cartesian plan, measuring the lengths of the sides, and computing the ratios. Using graphing calculators or computer software, as well as graphing simple examples by hand, have students graph a variety of trigonometric functions in which the amplitude, frequency, and/or midline is changed. Students should be able to generalize about parameter changes, such as what happens to the graph of y = sin(x) when the equation is changed to y = 2sin(x) + 5. Use graph paper and paper plate to model trigonometric functions Use teacher-guided comparison conversations to ensure that students are able to make connections Mathematics Journal Sample Prompts: • What patterns did you find in ….? • How do you …? • Review what you did today and explain how it is similar to something you already knew? • Is there a shortcut for finding …? What is it? How does it work? Why does it work? Emphasize multiple representations in teaching new and review older vocabulary: • Words • Algebraically • Tables of Data	**Formative Assessment** **LAUSD Assessments** The district will be using the SMARTER Balanced Interim Assessments. Teachers would use the Interim Assessment Blocks (IAB) to monitor the progress of students. Each IAB can be given twice to show growth over time. **State Assessments** California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. There is no assessment for Algebra 1. The 11th grade assessment will include ítems from Algebra 1, Geometry, and Algebra 2 standards. For examples, visit the SMARTER Balance Assessment at: http://www.smarterbalanced.org/ Sample Smarter Balanced Items: http://sampleitems.smarterbalanced.org/itempreview/sbac/index.htm

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
ng/high_school/trigonometry/gravel_roads.pdf	• Graphically • Symbolically Use a compass and straightedge to explore a unit circle with a fixed radius of 1. Help students to recognize that the circumference of the circle is 2π, which represents the number of radians for one complete revolution around the circle. Students can determine that, for example, π/4 radians would represent a revolution of 1/8 of the circle or 45°. Students can examine how a counterclockwise rotation determines a coordinate of a particular point in the unit circle from which sine, cosine, and tangent can be determined. Have students explore real-world examples of periodic functions; such as: average high (or low) temperatures throughout the year, the height of ocean tides as they advance and recede, and the fractional part of the moon that one can see on each day of the month. Graphing some real-world examples can allow students to express the amplitude, frequency, and midline of each.	

LANGUAGE GOALS for low achieving, high achieving, students with disabilities and English Language Learners
• Students will explain orally and in writing the attributes specific of a given quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, and identify whether the graph is [of] a circle or an a parabola. *Example:* "$3x^2 + x + y + 5 = 0$ is a parabola because the coefficient in front of y^2 is zero. • Students will listen for and point out similarities in their classmates' ideas using the words *similar*, *identical*, and *alike*. *Example:* _____'s ideas were similar/identical/alike to _____'s idea. • Students will explain the differences and similarities between trigonometric functions, using the following specific set of words: *sine*, *cosine*, *tangent*, *period*, and *amplitude*. • *Example*: " y = sin x and y = cos x graphs both have period of 360^{o} (2π) and an amplitude of 1"
PERFORMANCE TASKS
Illustrative Mathematics • As the Wheel Turns: F-TF.5, F-IF http://www.illustrativemathematics.org/illustrations/595

- Foxes and Rabbits 2: F-TF.5 http://www.illustrativemathematics.org/illustrations/816

Mathematics Assessment Project (MARS Tasks)

- The Ferris Wheel:F-TF.5 http://map.mathshell.org/materials/download.php?fileid=1252

NCTM Illuminations Lessons

- The Unit Circle: F-TF.1, 2 http://illuminations.nctm.org/LessonDetail.aspx?id=L785
- Hanging Chains: G-GPE.3.1 http://illuminations.nctm.org/LessonDetail.aspx?id=L628
- Rolling into Radians: http://illuminations.nctm.org/LessonDetail.aspx?id=L844
- Seeing Music: F-TF.5 http://illuminations.nctm.org/LessonDetail.aspx?id=L686
- Graphing Trigonometric Functions: F-TF.2.1 and F-TF.5 http://illuminations.nctm.org/ActivityDetail.aspx?ID=174

DIFFERENTIATION		
FRONT LOADING	ACCELERATION	INTERVENTION
In the extension of the trigonometric functions to the unit circle, proficient students must use repeated reasoning (MP.8). Students will model real world situations with trigonometric functions (MP.4). Use of trigonometric vocabulary, such as (amplitude, frequency, period, midline, degree, and radian) aid in communicating precisely (MP.6). **Pre-Teach Vocabulary** • State the definitions, and have students repeat the definitions. • Provide students with correct and incorrect usage of the word • Equation of Circles 1: http://map.mathshell.org/materials/download.php?fileid=1202	Students can investigate daylight hours model and many other trigonometric modeling situations such as simple predator-prey models, sound waves, and noise cancellation models. Engage students to use trigonometric functions to model periodic phenomena. Connected to standard F-BF.3 (families of functions), they begin to understand the relationship between the parameters appearing in the general cosine function $f(x)=A\cdot \cos(Bx-C)+D$ (and sine function) and the graph and behavior of the function (e.g., amplitude, frequency, line of symmetry).	Reteach the trigonometry ratio and remind students how to use (SOHCAHTOAH) to remember the trigonometric ratios. Teach students how to graph all 6 basic trigonometric functions, namely: sine, cosine, tangent, cotangent, secant, and cosecant. They can use any graphing utility such as graphing calculator, apps, and graphing software to graph the families of functions. Have students analyze and explain the meaning of amplitude, frequency, period, and midline based on their graphs.

References:

1. National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics)*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
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3. Engage NY. (2012). New York Common Core Mathematics Curriculum. Retrieved from http://engageny.org/sites/default/files/resource/attachments/a-story-of-ratios-a-curriculum-overview-for-grades-6-8.pdf.
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5. Smarter Balanced Assessment Consortium. (2012). Smarter Balanced Assessments. Retrieved from http://www.smarterbalanced.org/.
6. Partnership for Assessment of Readiness for College and Career. (2012). PARCC Assessments. Retrieved from http://www.parcconline.org/parcc-assessment.
7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
8. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx.
9. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions.

Algebra 2 – UNIT 5
Statistics and Probability

Critical Area:

Students analyze data to make sound statistical decisions based on probability models. By investigating examples of simulations of experiments and observing outcomes of the data, students gain an understanding of what it means for a model to fit a particular data set. Students develop a statistical question in the form of a hypothesis (supposition) about a population parameter, choose a probability model for collecting data relevant to that parameter, collect data, and compare the results seen in the data with what is expected under the hypothesis. Students build on their understanding of data distributions to help see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). In addition, they can learn through examples the empirical rule, that for a normally distributed data set, 68% of the data lies within one standard deviation of the mean, and that 95% are within two standard deviations of the mean.

CLUSTERS	COMMON CORE STATE STANDARDS
(s)Summarize, represent, and interpret data on a single count or measurement data.	**Statistics and Probability – Interpreting Categorical and Quantitative Data** **S.ID.4.** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
Understand and evaluate random processes underlying statistical experiments. **Make inferences and justify conclusions from sample surveys experiments, and observational studies.**	**Statistics and Probability – Making Inferences and Justifying Conclusions** **S.IC.1.** Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population. **S.IC.2.** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* **S.IC.3.** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **S.IC.4.** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **S.IC.5.** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. **S.IC.6.** Evaluate reports based on data**.**
Use probability to evaluate outcomes of decisions.	**Statistics and Probability – Using Probability to Make Decisions** **S.MD.6.** (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). **S.MD.7.** (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

MATHEMATICAL PRACTICES	
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. **Construct viable arguments and critique the reasoning of others.** 4. **Model with mathematics.** 5. **Use appropriate tools strategically.** 6. **Attend to precision.** 7. **Look for and make use of structure.** 8. Look for and express regularity in repeated reasoning.	Emphasize MP 1, 2, 3, 4, 5, 6, and 7 in this unit.

LEARNING PROGRESSIONS
Draft High School Progression on Statistics and Probability http://commoncoretools.me/wp-content/uploads/2012/06/ccss_progression_sp_hs_2012_04_21_bis.pdf

(m) Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.

(S) Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• In real life, data sets are large and almost always approximately normal. Normal models which include estimation of areas under the normal curve allow us to answer and model real life situations. • Sampling methods, when highly representative of a population, allow accurate predictions or inferences of population parameters. • Students model probabilities found in experimental environment and decide whether they are consistent with theoretical probabilities? • The mean or proportion of a sample is the same as the mean or proportion of a population, within a margin of error. • If the difference between the statistics of two	• Why do we study normal distributions? • Why is random sampling of a population done when a census is impractical? • Do experimental probabilities match theoretical probabilities? • How can a researcher select a method of collecting data with as little bias as possible? • How does the mean or proportion of a sample compare to the mean or proportion of the population? • When does a statistic become extraordinary instead of ordinary? • How do you know when the difference between two treatments is statistically significant.	Bell curve bias categorical data census complementary events conditional probability confidence interval convenience sample correlation coefficient counting methods critical value of z distribution experimental probability experimental study

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
treatments is outside of a critical confidence interval, the difference is statistically significant. • Select a method of gathering data from a random sample and understand data by critically differentiating the merit of reports and data encountered in daily life. • Probability can be used to develop strategies and make informed decisions.	• There are many "studies out there", how do I know if they are really accurate? • How can probability be used to make fair decisions?	fairness Histogram independence independent events margin of error mean (x-bar) normal model or normal distribution null hypothesis Numerical data observational study parameter population probability distribution proportion (p-hat) qualitative data random number generator random sample random variable representative sample sampling significant (as in statistics) simple random sample standard deviation statistic stratified random sample Subject survey systematic random sample theoretical probability treatment voluntary sample Z-Score

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
LAUSD Adopted Textbooks and Programs • Big Ideas Learning - Houghton Mifflin Harcourt, 2015: Big Ideas Algebra 2 • College Preparatory Mathematics, 2013: Core Connections, Algebra 2 • The College Board, 2014:Springboard Algebra 2 Materials: **California Revised Mathematics Framework:** http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp. **Illustrative Mathematics:** School Advisory Panel: S-IC.1 http://www.illustrativemathematics.org/illustrations/186 Strict Parents: S-IC.1, 3 http://www.illustrativemathematics.org/illustrations/122 Musical Preferences: S-IC.1, S-ID.5 http://www.illustrativemathematics.org/illustrations/123 SAT Score: S.ID.4 http://www.illustrativemathematics.org/illustrations/216 Do You Fit In This Car?: S.ID.4 http://www.illustrativemathematics.org/illustrations/1020	Engage students in a discussion or activity to clearly distinguish between categorical and numerical variables by providing multiple examples of each type. Have students practice their understanding of the different types of graphs for categorical and numerical variables by constructing statistical posters. Note that a bar graph for categorical data may have frequency on the vertical (student's sport preferences) or measurement on the vertical (students' grade in a test). One tool for developing statistical models is the use of simulations. This allows the students to visualize the model and apply their understanding of the statistical process. Provide students the opportunities to distinguish between a population parameter which is a constant, and a sample statistic which is a variable. Use teacher-guided comparison conversations to ensure that students are able to make connections. As the statistical process is being mastered by students, it is important for them to investigate questions such as "If a coin spun five times produces five tails in a row, could one conclude that the coin is biased toward tails?" Students will need to use all of the data analysis, statistics, and probability concepts covered to date to develop a deeper understanding of inferential reasoning. Have students critique published surveys before having them design their own surveys. Unlike	SBAC - http://www.smarterbalanced.org/ LAUSD ASSESSMENT The district will be using the SMARTER Balanced Interim Assessments. Teachers would use the Interim Assessment Blocks (IAB) to monitor the progress of students. Each IAB can be given twice to show growth over time. State Assessments California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. There is no assessment for Algebra 1. The 11th grade assessment will include ítems from Algebra 1, Geometry, and Algebra 2 standards. For examples, visit the SMARTER Balance Assessment at: http://www.smarterbalanced.org/ Sample Smarter Balanced Items: http://sampleitems.smarterbalanced.org/itempreview/sbac/index.htm

Should We Send Out a Certificate?: S.ID.4 http://www.illustrativemathematics.org/illustrations/1218	in observational studies; in surveys, the sample selected from a population needs to be representative of the population. Taking a random sample is generally what is done to satisfy this requirement. Use a variety of devices as appropriate to carry out simulations: number cubes, cards, random digit tables, graphing calculators, computer programs.	

LANGUAGE GOALS for **low achieving, high achieving, students with disabilities and English Language Learners**

- Students will describe orally and in writing the purposes of and differences among sample surveys, experiments, and observational studies.
  Example: "Based on the survey of teenage high school students, more students are more/less likely to _____ than _____."

- Students will decide whether a specified model is consistent with results from a data simulation.
  Example: "A model stating that a spinning coin falls heads up with a probability of 0.5 is not consistent with a simulation result of 5 tails in a row."

- Students will explain orally and in writing how they use statistical and probability concept in their lives, using the following specific set of words: *distribution*, *mean*, *standard deviation*, *probability*, and *statistics.*
  Example: "Based on the distribution of test scores with a mean of _____ and a standard deviation of _____, a test score of _____ is (*adjective).*
- Students will explain orally and in writing areas under the normal curve allow us to answer and model real life situations.

PERFORMANCE TASKS

Mathematics Assessment Projects (MARS Tasks)

- **Modeling Conditional Probabilities 1: Lucky Dip: S.MD.6** http://map.mathshell.org/materials/lessons.php?taskid=409&subpage=problem

NCTM Illuminations Lessons

- **Should We Send a Certificate?**: S.ID.4 http://www.illustrativemathematics.org/illustrations/1218
- **Exploration with Chance:** S.ID.6 http://illuminations.nctm.org/LessonDetail.aspx?id=L290

Illuminations
Fred's Fun Factory: S-MD.2, 5 and 7
http://www.illustrativemathematics.org/illustrations/1197
Miscellaneous Sources

- **The Normal Distribution: S.ID.4** http://www.wmich.edu/cpmp/1st/unitsamples/pdfs/C3U5_362-375.pdf
- **Applications of Probability:** http://www.schools.utah.gov/CURR/mathsec/Core/Secondary-II/II-4-S-MD-H-6-and-7.aspx

DIFFERENTIATION📖		
UDL/FRONT LOADING	ACCELERATION	INTERVENTION
Students should be encouraged to persevere when problem solving in this unit. Multiple solutions are common and should be recognized. Students can often make sense of complex contextual probabilities by considering a simpler analogous Probability situation (MP.1). As students work to identify events for which probabilities are to be determined and rules to apply, encourage students to verify and critique the thinking of their classmates (MP.3). Students have the opportunity to demonstrate proficiency with MP.6 by paying close attention to precise use of new vocabulary and writing complete sentences describing probabilities.	S.MD.7 Apply this standard with more complex probability models. You can implement the following activity: But mango is my favorite… http://www.illustrativemathematics.org/illustrations/1333 Often two sample groups are compared in clinical studies. Two key criteria are specified: are the data normally distributed and are the data paired? Unpaired (independent) normally distributed data: Student's unpaired two-sample t-test For example, the efficacy of a new drug A may be compared with an established drug B. The study has 220 patients in treatment Group A with sample mean x̄A and standard deviation SDA and 200 patients in treatment Group B with sample mean x̄B and standard deviation SDB; (Group A and Group B do not have to be equal). We need to calculate the difference between the two sample means and the standard error of this difference between the two means, from which we can calculate a confidence interval for the difference between them. For t-test to be valid, the standard deviations of both groups must be similar. This is often the case, even when the sample means are significantly different.	Review the difference between independent events and dependent events. Review the conversions of: • Ratios • Percentages • Decimals Teach students how to understand data in multiple forms: • Graphs • Charts • Table Review key vocabulary words from previous sections

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